A Rocket Projectile has the following characteristics

(1) Initial mass 200kg

Mass after Rocket operation 130kg

Payload, non Propulsive structure 110kg

Rocket operating duration 3.0 sec

Average specific impulse of Propellant 240sec

Determine the Vehicle's mass ratio, Propellant mass fraction, Propellant flow rate, thrust, thrust to weight ratio, acceleration of vehicle, effective exhaust velocity, total Impulse and the impulse-to weight ratio.

Solution:

(i) Mass ratio Vehicle $MR = m_f/m_o$

$$= 130/200 = 0.65$$

Mass ratio of Propulsion system $= MR = m_f/m_o$

$$= (130-110)/(200-110) = 0.222$$

(ii) Propellant mass fraction ξ

$$\xi = (m_o - m_f)/m_o = (90-20)/90$$

$$\xi = 0.778$$

(iii) Propellant flow rate $(\dot{m}) = m/s$

Propellant mass $200 - 130 = 70$ kg

$$\dot{m} = 70/3 = 23.3 \text{ kg/s}$$

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(iv) Thrust

$$F = I_s \dot{w} = 240 \times 23.3 \times 9.81$$

$$F = 54,857 \text{ N}$$

(v) Thrust to weight ratio:

Initial Value $F/w_0 = \dfrac{54,857}{200 \times 9.81} = 28$

Final Value $F/w_f = \dfrac{54,857}{(130 \times 9.81)} = 43$

(vi) Maximum acceleration of vehicle

$$43 \times 9.81 = 421 \text{ m/s}^2$$

(vii) Effective exhaust velocity

$$c = I_s g_0 = 240 \times 9.81 = 2354 \text{ m/s}$$

(viii) Total impulse (I_t)

$$I_t = I_s w_f = 240 \times 70 \times 9.81 = 164800$$

(ix) Impulse to weight ratio: ~~of Propellant~~

$$\frac{I_t}{w_0} = \frac{I_s}{(M_f + m_p) g_0} = \frac{164800}{(130 + 70) g_0} =$$

$$\frac{I_t}{w_0} = \frac{I_s}{m_p g_0} = \frac{164800}{(200 - 110) g_0} = 187 \text{ (Propellant alone)}$$

Rocket Equation:

The mass of a complete rocket vehicle consists of three parts (1) mass of payload M_L (2) mass of the vehicle structure M_S (3) mass of the propellants M_P.

$$M = M_L + M_S + M_P$$

Now consider the rocket blast off with zero at starting and shut down after the propellant burnes out at the velocity V_b. Now according to Newton's 2nd law $F=ma$

$$F = M\frac{dV}{dt} \quad \text{-----} \ (1)$$

Force F is equal to rocket engine force, the aerodynamic drag and weight of vehicle. Now taking thrust alone

$$T = M\frac{dv}{dt} \quad \text{-----} \ (2)$$

Thrust to specific impulse we know

$$\because I_s = \frac{F}{\dot{\omega}}$$

$$T = \dot{\omega} I_s = g_0 \dot{m} I_{sp} \quad \text{-----} \ (3)$$

where $\dot{m}$ is the mass flow of the propellant

M is changing with time due to decrease in M_P

$$\dot{m} = -\frac{dM_P}{dt} = -\frac{dM}{dt} \quad \text{-----} \ (4)$$

Sub (4) in (3)

$$T = -g_0 I_{s} \frac{dM}{dt} \quad \text{-----} \ (5)$$

Sub (5) in (2)

$$-g_0 I_{s} \frac{dM}{dt} = M \frac{dv}{dt}$$

(or)

$$-\frac{dM}{M} = \frac{dV}{g_0 I_s} \quad \text{----} \rightarrow (6)$$

Integrating (6) $V=0$ $M=M_0$ (Initial)

$V=V_b$ $M=M_f$ (burnout)

$$-\int_{M_0}^{M_f} \frac{dM}{M} = \int_{M_f}^{M_0} \frac{dM}{M} = \frac{1}{g_0 I_s} \int_0^{V_b} dv \Rightarrow \ln[M]_{M_f}^{M_i} = \frac{1}{g_0 I_s}[v]_0^{v}$$

$$= \ln M_i - \ln M_f = \frac{1}{g_0 I_s}[v]$$

$$\ln \frac{M_0}{M_f} = \frac{V_b}{g_0 I_{sp}}$$

$$\boxed{V_b = g_0 I_s \ln \frac{M_0}{M_f}}$$

(or)

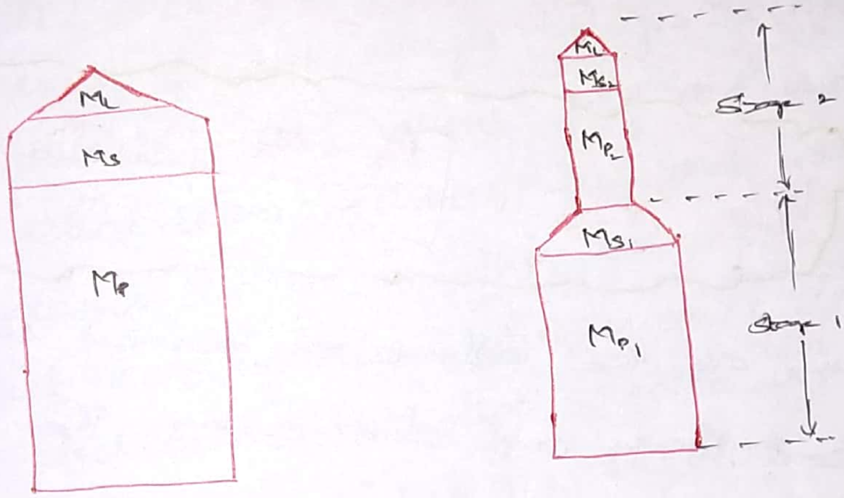
$$\boxed{\frac{M_i}{M_f} = e^{V_b (g_0 / I_s)}}$$

The above equation called as Rocket equation. Relates the burnout Velocity of a Rocket Vehicle

Rocket STAGING

Considering single stage Rocket & multi stage Rocket to explain which is better for placing Payload in the space

Let us first consider a single -stage Rocket. Where M_L - Mass of Payload, M_s mass of structure, & M_P mass of Propellant.



The burnout velocity of single stage Rocket is

$$V_b = g_0 I_{sp} \ln \frac{M_i}{M_f}$$

Where

$$M_i = M_P + M_S + M_L$$

$$M_f = M_S + M_L$$

Now considering two stage Rocket, where the 1st stage has M_{P_1} Propellant mass & M_{s_1} structural mass of stage one. Second stage has M_L Payload mass, M_{P_2} & M_{s_2}.

$$V_{b_1} = g_0 I_s \ln \frac{M_i}{M_f}$$

V_{b_1} is the burnout velocity of 1st stage

Here initial masses are

$$M_i = M_{p_1} + M_{s_1} + M_{p_2} + M_{s_2} + M_L$$

Final mass is the structure mass of the first stage plus the total mass of the second stage.

$$M_f = M_{s_1} + M_{p_2} + M_{s_2} + M_L$$

$$\boxed{V_{b_1} = g_0 I_{sp} \ln \left[\frac{M_{p_1} + M_{s_1} + M_{p_2} + M_{s_2} + M_L}{M_{s_1} + M_{p_2} + M_{s_2} + M_L} \right]}$$

First stage at the instant of burnout separates from the second stage and drop away. The rocket engine of the second stage ignites and boost the second stage from its initial velocity V_{b_1} to final velocity V_{b_2}

$$V_{b_2} - V_{b_1} = g_0 I_{sp} \ln \left(\frac{M_i}{M_f} \right)_2$$

Where

$$M_i = M_{p_2} + M_{s_2} + M_L$$

$$M_f = M_{s_2} + M_L$$

$$\boxed{V_{b_2} - V_{b_1} = g_0 I_{sp} \ln \frac{M_{p_2} + M_{s_2} + M_L}{M_{s_2} + M_L}}$$

consider the single-stage rocket and the double-stage rocket streched in Fig. Both the rockets have the same total mass $M_{total} = 5000$ kg and the same specific impulse $I_{sp} = 350$ s. Both rockets have the same pay load mass $M_L = 50$ kg. The total structural mass of the ~~single~~ double stage rocket is $M_{s1} + M_{s2} = 400$ kg $+ 100$kg $= 500$ kg, which is the structural mass of the single stage rocket. The total propellant mass of the double stage rocket is $M_{P1} + M_{P2} = 3450 + 1000 = 4450$ kg, which is the propellant mass of the single stage rocket. Both the rockets are boosting the same pay load mass of 50 kg into space. The breakdown between payload, structural, and propellant masses chosen in this example is purely arbitrary, but keeping the total masses in each category the same between the two rockets is intentional. In this way, the only difference between the rockets in Fig. is that one is a single-stage rocket and other is a double-stage rocket, but with the same total masses distributed over two stages. Calculate and compare the burnout velocities for the rockets.

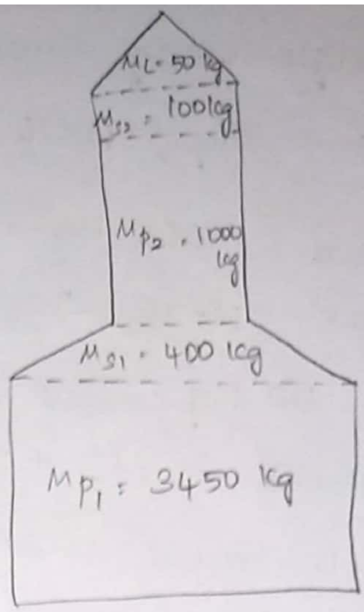
Sol:

For the single stage rocket in Fig a, the initial and final masses are

$$M_i = M_P + M_S + M_L = 4450 + 500 + 50 = 5000 \text{ kg}.$$

$$M_f = M_S + M_L = 500 + 50 = 550 \text{ kg}.$$

a) single-stage



b) Double stage.

From Eq. 9.51.

$$V_b = g_0 I_{sp} \cdot \ln \frac{M_i}{M_f} = 9.8\,(350)\,\ln \frac{5000}{550} = 7570 \text{ m/s}$$
$$= \cancel{7.5}\ 7.57 \text{ km/s}$$

For the double stage rocket in fig b, we have for the burnout velocity of the first stage from Eqn (9.56)

$$V_{b1} = g_0 I_{sp} \ln \frac{M_{p1} + M_{s1} + M_{p2} + M_{s2} + M_L}{M_{s1} + M_{p2} + M_{s2} + M_{PL}}$$

$$= 9.8\,(350)\,\ln \frac{3450 + 400 + 1000 + 100 + 50}{400 + 1000 + 100 + 50}$$

$$= 9.8\,(350)\,\ln \frac{5000}{1550} = 4017 \text{ m/s}.$$

The increase in velocity provided by the second stage is given by Eq. (9.60)

$$V_{b2} - V_{b1} = g_0 I_{sp} \ln \frac{M_{P2} + M_{S2} + M_L}{M_{S2} + M_L}$$

$$= 9.8\,(350) \ln \frac{1000 + 100 + 50}{100 + 50}$$

$$= 9.8\,(350) \ln \frac{1150}{150} = 6987 \text{ m/s}.$$

Hence the ~~burnout~~ velocity at burnout of the second stage is

$$V_{b2} = 6987 + V_{b1} = 6987 + 4017 = 11,004 \text{ m/s} = 11 \text{ km/s}.$$

Comparison:-

From this example, we see that the payload of 50 kg is launched into space at a velocity of 11 km/s by the double-stage rocket, where as for the same total expenditure of propellants, the single-stage rocket provides a velocity of only 7.57 km/s. Indeed, for this example, the single stage rocket